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PID control of second-order systems with hysteresis[†]

Bayu Jayawardhana*, Hartmut Logemann*, Eugene P. Ryan*

Abstract—The efficacy of proportional, derivative and integral (PID) control for set point regulation and disturbance rejection is investigated in a context of mechanical systems with hysteretic components. Two basic structures are studied: in the first, the hysteretic component resides (internally) in the restoring force action of the system (“hysteretic spring” effects); in the second, the hysteretic component resides (externally) in the input channel (e.g. piezo-electric actuators). In each case, robust conditions on the PID gains, explicitly formulated in terms of system data, are determined under which asymptotic tracking of constant reference signals and rejection of constant disturbance signals is guaranteed.

I. INTRODUCTION

We consider control of single-input (mechanical) systems of each of the following two forms:

$$m\ddot{x} + c\dot{x} + \Phi(x) = u + d, \quad (1)$$

$$m\ddot{x} + c\dot{x} + kx = \Phi(u + d_2) + d_1, \quad (2)$$

with control input $t \mapsto u(t) \in \mathbb{R}$ and constant (but unknown) disturbances $d, d_1, d_2 \in \mathbb{R}$. In a mechanical context, $x(t)$ represents displacement at time $t \in \mathbb{R}_+ := [0, \infty)$, m and c are the mass and the damping constant, and, in (2), k is a linear spring constant.

In the case of (1), the operator Φ models a restoring force which may exhibit hysteresis phenomena, a particular example of which is the “hysteric spring” model discussed in, for example, [17], [2]. In the case of (2), the operator Φ models hysteretic actuation. Such hysteretic effects arise in, for example, micro-positioning control problems using piezo-electric actuators or smart actuators, as investigated in, among other papers in this field, [1], [4], [5], [6], [8], [10], [15], [16], [18].

Motivated by a recent study in [7], for each of the above system structures we will investigate the efficacy of a PID controller of the form

$$u(t) = -k_p(x(t) - r) - k_d\dot{x}(t) - k_i \int_0^t (x(\tau) - r) d\tau + u_0, \quad (3)$$

where u_0 is the initial condition on the integrator, $k_p, k_d, k_i \geq 0$ are the controller gains and r is a constant reference signal to be tracked. The latter is a distinguishing feature

of the present paper vis à vis [7]. The investigation in [7] is concerned with systems of form (1) and is focussed on one particular hysteresis component, namely the Bouc-Wen model [19]. By contrast, in this paper we deal with a large class of rate-independent causal hysteresis operator Φ which includes the play operator, stop operator, backlash operator and Preisach operators. These operators are discussed in detail by Mayergoyz in [14], by Brokate and Sprekels in [3] and by Logemann and Mawby in [11].

The analytical framework for the present paper is provided by frequency-domain conditions developed in [12], [13] which ensure existence, regularity and appropriate asymptotic properties of solutions of a feedback interconnection of a linear (infinite dimensional) system and a hysteresis operator Φ . Within this framework and for each of the underlying system structures (1) and (2), robust design criteria – formulated explicitly in terms of bounds on the plant parameters m, c, k and on a Lipschitz constant associated with the hysteresis operator Φ – are developed under which disturbance rejection is assured and the tracking error $x(t) - r$ converges, as $t \rightarrow \infty$, to zero at exponential rate.

We conclude this introduction with some remarks on terminology and notation. As usual, we denote the space of continuous functions $I \rightarrow \mathbb{R}$, $I \subset \mathbb{R}$ an interval, by $C(I)$. A function $f \in C(\mathbb{R}_+)$ is said to be *piecewise monotone* if, for some strictly increasing unbounded sequence $(t_i)_{i=0}^\infty$ in \mathbb{R}_+ with $t_0 = 0$, f is monotone on $[t_{i-1}, t_i]$ for all $i \in \mathbb{N}$; the space of all such piecewise monotone functions is denoted by $C_{\text{pm}}(\mathbb{R}_+)$. The (Banach) space of measurable functions $f: \mathbb{R}_+ \rightarrow \mathbb{R}$ such that $\|f\|_{L^p} := \int_0^\infty |f(t)|^p dt < \infty$, $1 \leq p < \infty$, is denoted by $L^p(\mathbb{R}_+)$. For $f \in L^p(\mathbb{R}_+)$ and $T > 0$, f_T denotes the concatenation of the functions $f|_{[0, T]}$ and 0, given by

$$f_T(t) := \begin{cases} f(t), & t \in [0, T] \\ 0, & t \in (T, \infty). \end{cases}$$

The space $L^p_{\text{loc}}(\mathbb{R}_+)$ consists of all measurable functions $f: \mathbb{R}_+ \rightarrow \mathbb{R}$ such that $f_T \in L^p(\mathbb{R}_+)$ for all $T > 0$. By $W^{1,1}_{\text{loc}}(\mathbb{R}_+)$ we denote the space of locally absolutely continuous real-valued functions defined on \mathbb{R}_+ , that is, $f \in W^{1,1}_{\text{loc}}(\mathbb{R}_+)$ if and only if there exists $g \in L^1_{\text{loc}}(\mathbb{R}_+)$ such that $f(t) = f(0) + \int_0^t g(s) ds$ for all $t \in \mathbb{R}_+$.

A function $u \in C(\mathbb{R}_+)$ is *ultimately non-decreasing (non-increasing)* if there exists $\tau \in \mathbb{R}_+$ such that u is non-decreasing (non-increasing) on $[\tau, \infty)$; u is said to be *approximately ultimately non-decreasing (non-increasing)*, if for all $\varepsilon > 0$, there exists an ultimately non-decreasing (non-

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increasing) function $v \in C(\mathbb{R}_+)$ such that

$$|u(t) - v(t)| \leq \varepsilon \quad \forall t \in \mathbb{R}_+.$$

II. HYSTERESIS OPERATORS

An operator $\Psi : C(\mathbb{R}_+) \rightarrow C(\mathbb{R}_+)$ is said to be *causal* if, for all $\tau \geq 0$ and all $v_1, v_2 \in C(\mathbb{R}_+)$, $v_1 = v_2$ on $[0, \tau]$ implies that $\Psi(v_1) = \Psi(v_2)$ on $[0, \tau]$.

A function $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a *time transformation* if f is continuous and non-decreasing and $\lim_{t \rightarrow \infty} f(t) = \infty$. An operator $\Phi : C(\mathbb{R}_+) \rightarrow C(\mathbb{R}_+)$ is *rate independent* if, for every time transformation f ,

$$(\Phi(u \circ f))(t) = (\Phi(u))(f(t)) \quad \forall u \in C(\mathbb{R}_+), t \in \mathbb{R}_+.$$

The operator $\Phi : C(\mathbb{R}_+) \rightarrow C(\mathbb{R}_+)$ is said to be a *hysteresis operator* if Φ is causal and rate independent.

The *numerical value set*, NVS Φ , of a hysteresis operator Φ is defined by

$$\text{NVS } \Phi := \{(\Phi(u))(t) \mid u \in C(\mathbb{R}_+), t \in \mathbb{R}_+\}.$$

For $w \in C([0, \alpha])$ (with $\alpha \geq 0$) and $\gamma, \delta > 0$, we define

$$\mathcal{C}(w; \delta, \gamma) := \{v \in C([0, \alpha + \gamma]) \mid v|_{[0, \alpha]} = w, \max_{t \in [\alpha, \alpha + \gamma]} |v(t) - w(\alpha)| \geq \delta\}.$$

We will have occasion to impose some or all of the following conditions on the hysteresis operator $\Phi : C(\mathbb{R}_+) \rightarrow C(\mathbb{R}_+)$:

- (N1) If $u \in W_{\text{loc}}^{1,1}(\mathbb{R}_+)$, then $\Phi(u) \in W_{\text{loc}}^{1,1}(\mathbb{R}_+)$;
 (N2) The operator Φ is monotone in the sense that, if $u \in W_{\text{loc}}^{1,1}(\mathbb{R}_+)$, then

$$(\Phi(u))'(t)u'(t) \geq 0, \quad \text{a.e. } t \in \mathbb{R}_+;$$

- (N3) There exists $\lambda > 0$ such that for all $\alpha \geq 0$ and $w \in C([0, \alpha])$, there exist constants $\gamma, \delta > 0$ such that

$$\begin{aligned} \max_{\tau \in [\alpha, \alpha + \gamma]} |(\Phi(u))(\tau) - (\Phi(v))(\tau)| \\ \leq \lambda \max_{\tau \in [\alpha, \alpha + \gamma]} |u(\tau) - v(\tau)| \\ \forall u, v \in \mathcal{C}(w; \delta, \gamma); \end{aligned}$$

- (N4) For all $\alpha \in \mathbb{R}_+$ and all $u \in C([0, \alpha])$, there exist $c > 0$ such that

$$\max_{\tau \in [0, t]} |(\Phi(u))(\tau)| \leq c(1 + \max_{\tau \in [0, t]} |u(\tau)|) \quad \forall t \in [0, \alpha];$$

- (N5) If $u \in C(\mathbb{R}_+)$ is approximately ultimately non-decreasing and $\lim_{t \rightarrow \infty} u(t) = \infty$, then $(\Phi(u))(t)$ and $(\Phi(-u))(t)$ converge, as $t \rightarrow \infty$, to $\sup \text{NVS } \Phi$ and $\inf \text{NVS } \Phi$, respectively;

- (N6) If, for $u \in C(\mathbb{R}_+)$, $\lim_{t \rightarrow \infty} (\Phi(u))(t) \in \text{int}(\text{NVS } \Phi)$, then u is bounded.

These technical assumptions underpin the proofs (which can be found in [9]) of Theorems 3.1 and 3.3 below; moreover, they are natural in the sense that they hold for the most

commonly-encountered hysteresis operators: relay, elastic-plastic, backlash, Prandtl, Preisach. Furthermore, we remark that many hysteresis operators (see, for example, [3], [11]) are *Lipschitz continuous* in the sense that

$$\begin{aligned} \sup_{\tau \in \mathbb{R}_+} |(\Phi(u))(\tau) - (\Phi(v))(\tau)| \\ \leq \lambda \sup_{\tau \in \mathbb{R}_+} |u(\tau) - v(\tau)| \quad u, v \in C(\mathbb{R}_+), \end{aligned} \quad (4)$$

for some $\lambda > 0$, in which case (N3) is (trivially) satisfied and, furthermore, (N1) holds (see [11]). In the next subsection, we briefly digress to describe the backlash and Preisach operators which are widely adopted as hysteresis models in engineering applications.

A. Backlash, Prandtl and Preisach operators

1) *Backlash operator*: The backlash (or play) operator, widely used in mechanical models (of, for example, gear trains or of hydraulic servovalves), has been discussed rigorously in many references, see for example [3], [11], [14]. With a view to giving a precise definition of backlash, we first define, for each $h \in \mathbb{R}_+$, the function $b_h : \mathbb{R}^2 \rightarrow \mathbb{R}$ by

$$b_h(v, w) := \max\{v - h, \min\{v + h, w\}\}.$$

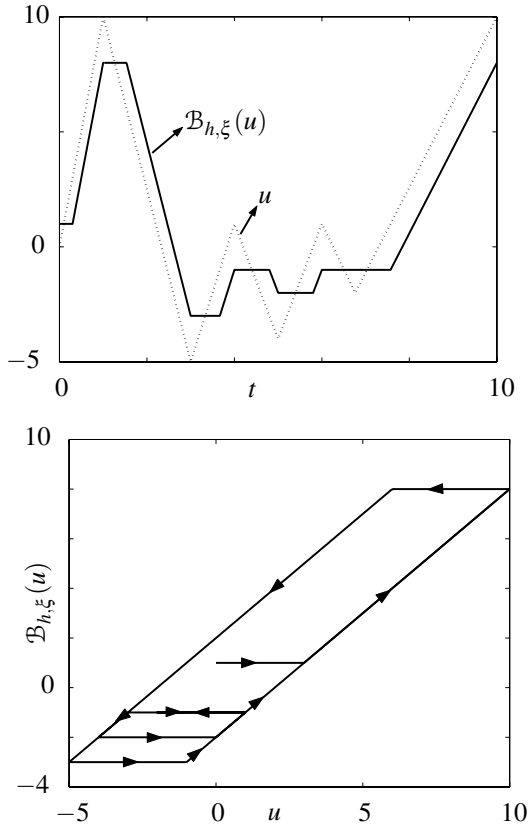
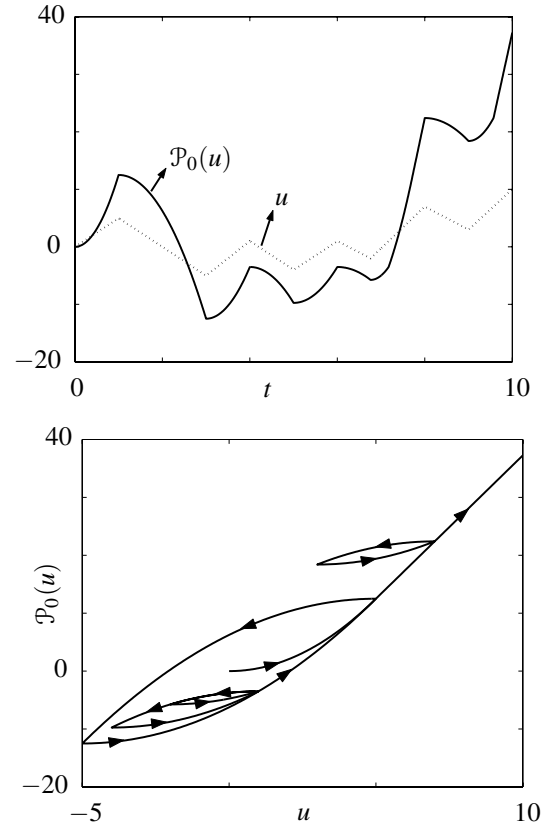
For all $h \in \mathbb{R}_+$ and all $\xi \in \mathbb{R}$, we introduce an operator $\mathcal{B}_{h, \xi}$ defined on the space $C_{\text{pm}}(\mathbb{R}_+)$ of piecewise monotone functions, by defining, for every $u \in C_{\text{pm}}(\mathbb{R}_+)$,

$$\left. \begin{aligned} (\mathcal{B}_{h, \xi}(u))(0) &:= b_h(u(0), \xi) \\ (\mathcal{B}_{h, \xi}(u))(t) &:= b_h(u(t), (\mathcal{B}_{h, \xi}(u))(t_i)) \\ &\quad t \in (t_{i-1}, t_i], i \in \mathbb{N} \end{aligned} \right\} \quad (5)$$

where $0 = t_0 < t_1 < t_2 < \dots$ is a partition of \mathbb{R}_+ , such that u is monotone on each of the intervals $[t_{i-1}, t_i]$, $i \in \mathbb{N}$. Here ξ plays the role of an “initial state”. It is well known, see, for example, [3, page 42], that the operator $\mathcal{B}_{h, \xi} : C_{\text{pm}}(\mathbb{R}_+) \rightarrow C(\mathbb{R}_+)$ can be extended uniquely to a hysteresis operator $\mathcal{B}_{h, \xi} : C(\mathbb{R}_+, \mathbb{R}) \rightarrow C(\mathbb{R}_+, \mathbb{R})$; moreover, the extended operator is Lipschitz continuous (in the sense of (4)) with Lipschitz constant $\lambda = 1$ and satisfies (N1)–(N6) (see, for example, [11, Proposition 5.4]). The action of a backlash operator is illustrated in Figure 1.

2) *Prandtl and Preisach operators*: The Preisach operator, a version of which is described below, encompasses backlash and represents a far more general type of hysteresis which, for certain input functions, exhibits nested loops in the corresponding input-output characteristics. Let $\zeta : \mathbb{R}_+ \rightarrow \mathbb{R}$ be a compactly supported and globally Lipschitz function with Lipschitz constant 1. Let $w : \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}$ be locally essentially bounded and let $\alpha : \mathbb{R}_+ \rightarrow \mathbb{R}$ be locally integrable. The hysteresis operator $\mathcal{P}_\zeta : C(\mathbb{R}_+) \rightarrow C(\mathbb{R}_+)$ defined by

$$\begin{aligned} (\mathcal{P}_\zeta(u))(t) \\ = \int_0^\infty \int_0^{(\mathcal{B}_{h, \zeta(h)}(u))(t)} w(s, h) \alpha(h) ds dh + w_0, \\ \forall u \in C(\mathbb{R}_+), \forall t \in \mathbb{R}_+, \end{aligned} \quad (6)$$

Fig. 1. Backlash operator $\mathcal{B}_{h,\xi}$ ($h = 2$, $\xi = 1$)Fig. 2. Prandtl operator (8) with $\lambda = 5$.

is called a *Preisach* operator (see, for example [3, p.55]). Under the assumptions that (i) both α and w are non-negative valued, (ii) α is in $L^1(\mathbb{R}_+)$ and (iii) w is essentially bounded (with norm $\|w\|_{L^\infty}$), then the operator \mathcal{P}_ζ is Lipschitz continuous with Lipschitz constant $\lambda = \|\alpha\|_{L^1}\|w\|_{L^\infty}$ and, as shown in [11], (N1)–(N6) hold.

The special case wherein $w \equiv 1$ and $w_0 = 0$ in (6), yields the *Prandtl* operator:

$$(\mathcal{P}_\zeta(u))(t) = \int_0^\infty (\mathcal{B}_{h,\zeta(h)}(u))(t) \alpha(h) dh, \quad \forall u \in C(\mathbb{R}_+) \quad \forall t \in \mathbb{R}_+. \quad (7)$$

For example, if $\alpha = \chi_{[0,l]}$, where $\chi_{[0,l]}$ is the indicator function of $[0,l]$, and $\zeta = 0$, then we have the Prandtl operator given by

$$\begin{aligned} (\mathcal{P}_0(u))(t) &= \int_0^\infty \chi_{[0,l]}(h) (\mathcal{B}_{h,0}(u))(t) dh \\ &= \int_0^l (\mathcal{B}_{h,0}(u))(t) dh. \end{aligned} \quad (8)$$

This operator satisfies (N3) with $\lambda = l$, has numerical value set $\text{NVS } \mathcal{P}_0 = \mathbb{R}$, and exhibits nested loops as depicted in Figure 2. We will revisit this hysteresis operator in Section IV below.

III. PID CONTROL OF SYSTEMS WITH HYSTERESIS

We now focus attention on the analysis and design of PID control in the context of each of system structures (1) and (2).

A. Systems of form (1)

Consider again a second-order system described by (1):

$$\left. \begin{aligned} m\ddot{x} + c\dot{x} + \Phi(x) &= u + d, \\ x(0) &= x_0, \quad \dot{x}(0) = v_0, \\ m, c, d &\in \mathbb{R}, \quad m > 0, \quad c > 0, \end{aligned} \right\} \quad (9)$$

where d is a constant disturbance signal. Assume that $r \in \mathbb{R}$ is a constant reference signal, in which case, the control objective is to determine, by feedback, the control input u to achieve the tracking objective: $x(t) \rightarrow r$ as $t \rightarrow \infty$. We will investigate the efficacy of the following PID control in achieving this objective:

$$\begin{aligned} u(t) &= -k_p(x(t) - r) - k_d\dot{x}(t) \\ &\quad - k_i \int_0^t (x(\tau) - r) d\tau + u_0, \end{aligned} \quad (10)$$

where $u_0 \in \mathbb{R}$ is the initial condition of the integrator and $k_p, k_d, k_i \geq 0$ are suitably chosen gains.

Theorem 3.1: Let $\Phi : C(\mathbb{R}_+) \rightarrow C(\mathbb{R}_+)$ be a hysteresis operator satisfying (N1), (N2) and (N3) (with associated

constant $\lambda > 0$). Let $r, d \in \mathbb{R}$. If k_p, k_d, k_i are chosen such that

- (A1) $k_d > -c + \sqrt{(2k_p + \lambda)m}$,
 (A2) $0 < k_i < k_p(k_p + \lambda)/(2(c + k_d))$,

then, for each $(x_0, v_0, u_0) \in \mathbb{R}^3$, the initial-value problem given by (9) and (10) has a unique solution $x \in C^2(\mathbb{R}_+)$, $x(t) \rightarrow r$, $\dot{x}(t) \rightarrow 0$ and $\ddot{x}(t) \rightarrow 0$ as $t \rightarrow \infty$; moreover, $(\Phi(x))(t)$ converges to a finite limit as $t \rightarrow \infty$ (all convergences being exponentially fast).

PROOF. Length restrictions preclude the inclusion of a proof of this result here. However, a proof can be found in [9], the essence of the proof is to restructure the initial-value problem (9)-(10) into a feedback form to which Theorem 4.1 of [12] may be applied. \square

Remark 3.2: Assume that the parameters m, c and λ are unknown, but belong to known intervals, viz. $m \in (0, m_+]$, $c \in [c_-, c_+]$ and $\lambda \in (0, \lambda_+]$, where $m_+ > 0$, $0 < c_- \leq c_+$ and $\lambda_+ > 0$ are known constants. If the PID controller gains are determined by using the following procedure:

- (P1) let $k_p > 0$ be arbitrary,
 (P2) choose k_d such that $k_d > -c_- + \sqrt{(2k_p + \lambda_+)m_+}$,
 (P3) choose k_i such that $0 < k_i < k_p^2/(2(c_+ + k_d))$,

then (A1) and (A2) hold and Theorem 3.1 applies to conclude that the PID controller, with the above choice of gain, solves the tracking and disturbance rejection problem \diamond

B. Systems of form (2)

Now consider a second-order system described by (2):

$$\left. \begin{aligned} m\ddot{x} + c\dot{x} + kx &= \Phi(u + d_2) + d_1, \\ x(0) &= x_0, \quad \dot{x}(0) = v_0, \\ m, c, k, d_1, d_2 &\in \mathbb{R}, \quad m, k, c > 0, \end{aligned} \right\} \quad (11)$$

where d_1 and d_2 are constant disturbance signals. Assume that $r \in \mathbb{R}$ is a constant reference signal. We will investigate the efficacy of the control structure (10) in both the absence ($k_d = 0$) and presence ($k_d > 0$) of derivative feedback.

Theorem 3.3: Let $\Phi : C(\mathbb{R}_+) \rightarrow C(\mathbb{R}_+)$ be a hysteresis operator satisfying (N1)–(N6) with associated $\lambda > 0$. Let $r, d_1, d_2 \in \mathbb{R}$ and assume that $rk - d_1 \in \overline{\text{NVS}}\Phi$.

Case (a). Set $k_d = 0$ and let $k_p, k_i > 0$ be chosen such that:

- (A) $0 < k_i < \frac{k_p k}{c} < \frac{ck}{\lambda m}$.

Case (b). Let $k_p, k_i, k_d > 0$ be chosen such that:

- (B1) $0 < k_i < \infty$,
 (B2) $k_p > \frac{ck_i}{k}$,
 (B3) $k_d > \frac{mk_p}{c}$.

Then there exists a unique solution $x \in C^2(\mathbb{R}_+)$ of the closed-loop system given by (10) and (11) such that

$$\lim_{t \rightarrow \infty} x(t) = r, \quad \lim_{t \rightarrow \infty} \dot{x}(t) = 0,$$

$$\lim_{t \rightarrow \infty} \ddot{x}(t) = 0, \quad \lim_{t \rightarrow \infty} (\Phi(u + d_2 \theta))(t) = rk - d_1.$$

Moreover, if $rk - d_1$ is an interior point of $\text{NVS}\Phi$, then the control signal u given by (10) is bounded.

PROOF. Again, length restrictions preclude the inclusion of a proof of this result here. However, a proof can be found in [9], the essence of the proof is to restructure the initial-value problem (9)-(10) into a feedback form to which Theorem 4.1 of [13] may be applied. \square

Remark 3.4: (i) Assume that the parameters m, c, k and λ are unknown, but belong to known intervals:

$$m \in (0, m_+], \quad c \in [c_-, c_+], \quad k \in [k_-, \infty) \text{ and } \lambda \in (0, \lambda_+]$$

where m_+, c_-, c_+, k_- and λ_+ are known positive constants. We give procedures for choosing the PID-controller gains in terms of the constants m_+, c_-, c_+, k_- and λ_+ .

Case (a). Set $k_d = 0$. If the gains k_p, k_i of the PI controller are determined by the following procedure:

(PA) choose k_p, k_i such that

$$0 < k_i < \frac{k_p k_-}{c_+} < \frac{c_-^2 k_-}{\lambda_+ m_+ c_+},$$

then (A) holds and Theorem 3.3 applies to conclude that the PI controller, with the above gain selection, solves the tracking and disturbance rejection problem.

Case (b). In the case of PID control, if the controller gains are determined by the following procedure:

(PB1) choose $0 < k_i < \infty$,

(PB2) choose $k_p > 0$ such that $k_p > c_+ k_i / k_-$,

(PB3) choose $k_d > 0$ such that $k_d > m_+ k_p / c_-$,

then (B1), (B2) and (B3) hold and Theorem 3.3 applies to conclude that the PID controller, with the above gain selection, solves the tracking and disturbance rejection problem.

(ii) In general d_1 is unknown (as is d_2), but it is reasonable to assume that $d_1 \in [d_1^-, d_1^+]$, where d_1^- and d_1^+ are known. Moreover, it is reasonable to assume that $k \in [k_-, k_+]$, where $0 < k_- < k_+$ are known constants. The conditions

$$rk_- - d_1^+, rk_+ - d_1^- \in \overline{\text{NVS}}\Phi, \quad \text{if } r \geq 0,$$

$$rk_+ - d_1^+, rk_- - d_1^- \in \overline{\text{NVS}}\Phi, \quad \text{if } r < 0$$

are sufficient conditions in terms of d_1^-, d_1^+, k_- and k_+ guaranteeing that $rk - d_1 \in \overline{\text{NVS}}\Phi$ for all $d_1 \in [d_1^-, d_1^+]$ and all $k \in [k_-, k_+]$. \diamond

IV. EXAMPLE

In this section, we illustrate our main results in the context of a Prandtl hysteresis operator, as discussed in Section II-A. In particular, we consider the hysteresis operator $\Phi = \mathcal{P}_0$, defined by (8), with parameter $l > 0$, in which case, (N1)–(N6) hold with $\lambda = l$ in (N3) and $\text{NVS}\Phi = \mathbb{R}$.

A. System of form (1)

Consider system (9) with $\Phi = \mathcal{P}_0$ and with $m \in (0, 2]$, $c \in [1, 3]$ and $l = \lambda \in (0, 10]$. Assume a constant disturbance $d = 1$, reference signal $r = 1$ and zero initial condition $u_0 = 0$ on the controller integrator. Using the procedure in Remark 3.2, the gains of PID controller are chosen as follows: $k_p =$

$10, k_d = 8, k_i = 4$. For nominal plant parameters values $m = 1$, $c = 2$ and $\lambda = 5$, Figure 3 shows the evolution of the closed-loop system with zero initial state.

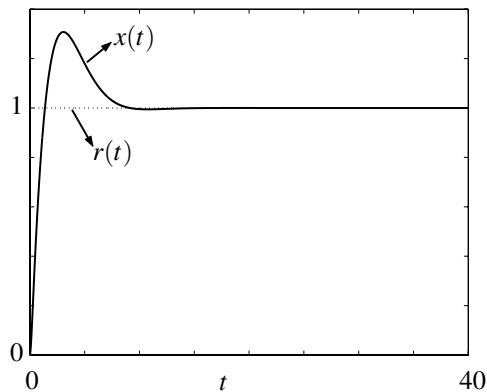


Fig. 3. System (1) under PID control.

B. System of form (2)

Finally, consider system (11) with Φ as above and with $m \in (0, 2]$, $c \in [1, 3]$, $\lambda \in (0, 10]$ and $k \geq 4$. Assume a constant disturbance $d_1 = 1$, reference signal $r = 1$, zero disturbance $d_2 = 0$ and $u_0 = 0$.

Case (a). Set $k_d = 0$. Using the procedure in Case (A) of Remark 3.4, the gains of PI controller are chosen as follows: $k_p = 0.04$, $k_i = 0.05$. For nominal plant parameters values $m = 1$, $c = 2$, $k = 4$ and $\lambda = 5$, Figure 4 shows the evolution of the closed-loop system with zero initial state: as $t \rightarrow \infty$, $x(t)$ converges (albeit slowly) to the constant reference signal r as predicted by Theorem 3.3.

Case (b). We now include derivative feedback action ($k_d > 0$). In this case, a PID controller is used instead of PI controller. Using the procedure in Case (B) of Remark 3.4, the gains of PID controller may be chosen as follows: $k_i = 10$, $k_p = 10$ and $k_d = 30$. Again, with nominal plant parameter values $m = 1$, $c = 2$, $k = 4$ and $\lambda = 5$, Figure 4 shows the evolution of the closed-loop system with zero initial state. It can be seen from this figure that, although the displacement $x(t)$ converges asymptotically to the constant reference signal r under either PI or PID control, the PID controller generates the fastest response: this is to be expected insofar as derivative feedback action is included.

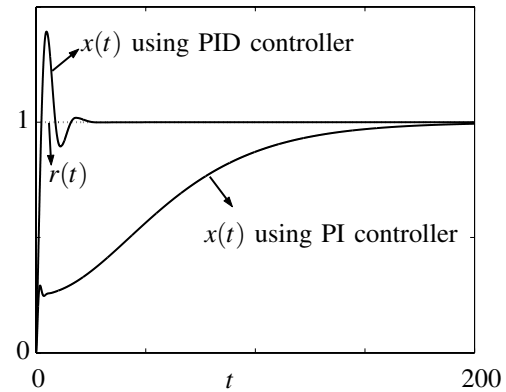


Fig. 4. System (2) under PI and PID control.

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